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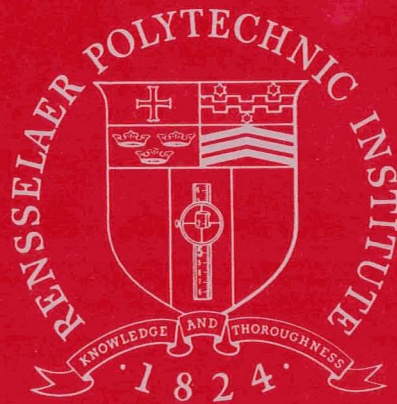
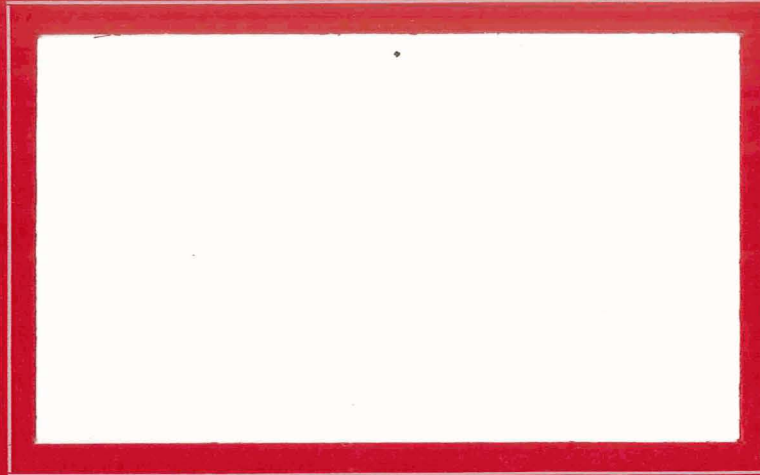
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PRELIMINARY DESIGN OF AN AUTOMATIC  
DEVICE FOR THE LOCATION OF THE POLE  
STAR AND/OR TRUE POLE OF MARS

by

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Analysis and Design of a Capsule Landing System  
and Surface Vehicle Control System for Mars Exploration

May 1971

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## ABSTRACT

For surface navigation of an unmanned vehicle on the planet Mars, the location of the Martian pole star can be used to help generate the needed reference frames. Since an unmanned vehicle's initial surface orientation is unknown, there would be some difficulty in locating the pole star. The object of this project is to design an automatic system for the unmanned vehicle whose purpose is to locate the pole star of Mars and to supply this information to the navigational equipment.

The automatic system operates on the concept that if a unique group of stars can be found in the heavens, a geometrical relationship can be derived from this group that allows the generation of the location of the pole star. This is accomplished through the use of a pattern recognition system. The system designed in this project operates on this principle and will locate the pole star and/or any point in the heavens desired. The pole star location generated by the system is more than accurate enough for most navigational needs.



## A. THE CONCEPT FOR AUTOMATIC LOCATION OF THE MARTIAN POLE STAR

### 1. Introduction

One of the primary difficulties encountered with a surface navigation system is the establishment of the proper reference frames. In the present navigation system, the local vertical and the pole star of Mars<sup>[1]</sup> are being used to generate the necessary coordinate systems. I am presently attempting to design an automatic system for the location of the pole star.

It should be mentioned that there is nothing special about the pole star. It is being used strictly for convenience and accuracy. Any other star could be used to locate a point in the sky that is over the pole. The heavens appear to rotate about a point in the sky over Mars' pole with stars and constellations traversing circular paths about this point. Picking any star, it is only necessary to have a clock and a chart of the star's motion to be able to calculate the point in the heaven that is over the pole, but this isn't too accurate and the accuracy decreases with increasing distance between the star and the point. This illustrates the advantages of using the pole star for the reference star, i.e., no accurate clock or astronomical chart for Mars is necessary, no inaccuracies from sightings and calculations are introduced, and implementation will probably be easier.

### 2. Basic Concept for Location of the Pole Star

An automatic pattern recognition system will be used to locate the pole star of Mars. This system will discriminate between the magnitudes and emitted wavelengths of the light emitted by the stars that it scans and it will use this information to pick the pole star directly

if the pole star is unique in character for the star group scanned. If the pole star isn't unique, a unique group of stars will be used for which there exists a geometrical relation that locates the pole star.

### 3. Basic Pattern Recognition Operations

In locating the pole star, there are two main operations that most pattern recognition systems would perform on the image that is received from the heavens.<sup>[2][3][4][5]</sup> The first is mapping the original image space back upon itself, thus allowing for translation of the image, orientation changes, scale changes and/or a reduction of background clutter. The second is mapping the original image space into a new space, meaning mapping into a spatial frequency plane or, into an abstract feature representation space or, into whatever space might be derived. Once these operations are completed (if they are needed at all), preprocessing techniques are used to obtain the needed information contained in the image. There are two basic approaches to preprocessing used in pattern recognition. The first is electronic processing which includes such operations as the processing of video gray scale information, Fourier Frequency analysis and statistical sampling. The second is optical processing including such operations as image enhancement using spatial filtering and using conventional power spectra and slit-aperture power spectra. Whether or not any or all of the above operations are necessary will become obvious as this report progresses.

#### B. IDENTIFICATION OF THE POLE STAR AND THE APPROACH FOR ITS LOCATION

##### 1. Determination of Celestial Coordinates of the Pole of Mars

As previously stated, the proposed navigation system requires the local vertical and the Martian pole star to generate the needed

coordinate systems. The first problem encountered in attempting to design an automatic system to locate the pole star was identifying the star.

The equator of Mars is inclined from the vertical of its orbit by an amount that is almost the same as the inclination of the equator of Earth to its orbit ( $24.936^\circ$  for Mars, as compared to  $23.5^\circ$  for Earth). Knowing that the axis of Earth points to within one degree of the star Polaris is of little help to us, for though both planets (Mars and Earth) have approximately the same inclination to their orbits, their axis point in different directions.<sup>[6]</sup> These directions differ by approximately  $45^\circ$ .

In the past, it has been found that the celestial coordinates of the north pole of Mars are needed to perform astronomical calculations concerning Mars. The methods used in determining the direction of the axis are generally based on one of three techniques:

- (1) Observations of the polar caps, with allowance for the slight eccentricity of the caps with respect to the areographic poles (Lowell 1905, Wortz 1912, Widorn 1939)
- (2) Determination of the axis of ellipses described by the rotation of selected surface markings near the equator (Trumpler 1927, Carmichel 1954)
- (3) Determination of the pole of the nearly equatorial orbits of the Martian satellites (Struve 1911, Burton 1929)

The results of these different methods are summarized in Table I and

the most representative figures, as determined by DeVaucouleur,<sup>[7]</sup> are

right ascension  $\alpha_0 = 316.55^\circ$  (1905.0)

declination  $\delta_0 = 452.85^\circ$

These values of right ascension and declination vary with time due to the precession and nutation of Mars. Because of this, the values are referred to a standard equinox (in this case 1905.0) for easy comparison. For the degree of precision usually required, nutation may be neglected. Compensating for precession, DeVaucouleur arrived at the compensated values of

$$\alpha_0(t) = 316.55^\circ + 0.00675(t-1905.0)$$

$$\delta_0(t) = 452.85^\circ + 0.00346(t-1905.0)$$

where  $|t-1905.0| < 100$  years

Thus we can locate the celestial coordinates of Mars axis for any time desired and these coordinates correspond to the coordinates of the pole star of Mars.

## 2. Identification of the Pole Star

Checking the stellar maps, it is found that there is no star directly over the pole of Mars. The nearest bright star, which would be considered the north pole star of Mars, is  $\alpha$  Cygnus (Deneb) which is within  $10^\circ$  of the exact polar location<sup>[8]</sup>. Now that the pole star is identified, it is necessary to find a method to locate it.

## 3. Mathematical Approach to Location of Pole Star and/or True Pole

Since there is nothing special about the pole star (meaning that any star could be used for the same purpose) and since Deneb is not directly over the pole, a method must be found for locating the pole star that could be used to locate any other star or to locate a

point in the heavens that is directly over the true pole. Deneb is bright, having a magnitude of 1.3 emitted at a spectral value of A2p, and it could be located directly but this would require too exacting a system since it would have to discriminate from other stars in the heavens with similar magnitude and spectra.

The method to be used requires the location of two specific distinct stars and using these two stars to generate the location that is desired, whether it be the pole star or a point in the heavens exactly over the pole. The location of these two stars is greatly facilitated if they are distinct in some manner from the other stars in the heavens. A study of maps of the northern celestial hemisphere indicate that most stars have other stars similar to them present in the heavens<sup>[6][8]</sup>. This led to the decision to use binary stars as the reference stars since they comprise a very small and unique set in the heavens, having a magnitude of emitted light and two distinct spectra (see Appendix I). Once these two binary stars are located, the pole star is found by swinging two arcs to specified angles with respect to the directed arc between the two reference stars. One arc originates from each reference star and their intersection is the location of the pole star (shown in Fig. 1)

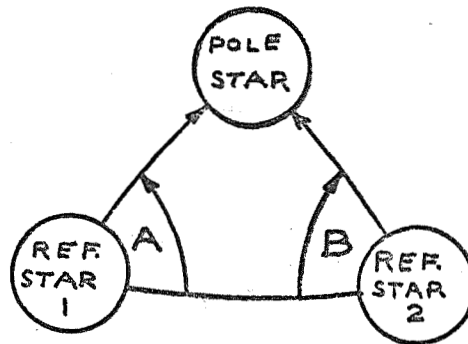


Figure 1

Picking Cygnus(Albireo and  $\delta$  Sagitta as the reference binary stars, the needed information on them was obtained.

	<u>Magnitude</u>	<u>Spectra</u>	<u>Right Ascension</u>	<u>Declination</u>
Deneb	1.3	A2p	311° 3.6'	45° 10' 35"
Albireo	3.2	K0,A0	293° 34.2'	27° 53' 51"
$\delta$ Sagitta	3.8	M0,A0	296° 43.44'	18° 27' 42"

To find angles A and B, in Fig. 2, it is necessary to find the arc lengths between the three stars and then to use these lengths, along with the proper spherical trigonometric [9] relations. Spherical trigonometry is also needed to find these original arc lengths since the location of each star is given in terms of its right ascension and declination measurements on the celestial sphere. In applying this type of trigonometry it must be remembered that it was derived for great circles and care must be taken to use only great circles; otherwise the relations are invalid.

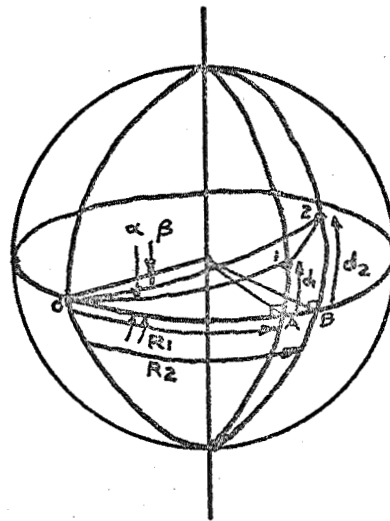


Figure 2

In general, to find the arc length between stars located at points 1 and 2, given their right ascensions and declinations ( $R_1, d_1$ )

and (R2,d2), one can proceed as follows:  $\gamma = 90^\circ$

arc 01 (1)  $\cos 01 = \cos R_1 \cos d_1 + \sin R_1 \sin d_1 \cos A$

$$\sin 01 / \sin A = \sin 01 / 1 = \sin d_1 / \sin \alpha$$

(2)  $\sin \alpha = \sin d_1 / \sin 01$

arc 02 (3)  $\cos 02 = \cos R_2 \cos d_2 + \sin R_2 \sin d_2 \cos B$

$$\sin 02 / \sin B = \sin 02 / 1 = \sin d_2 / \sin \beta$$

(4)  $\sin \beta = \sin d_2 / \sin 02$

since we are measuring  $\alpha$  and  $\beta$  at the equator  $\gamma = \beta - \alpha$

$$\cos 12 = \cos 01 \cos 02 + \sin 01 \sin 02 \cos \gamma$$

(5)  $\text{arc } 12 = \cos^{-1} 12$

Using the right ascensions and declinations for Deneb, Albireo and

$\delta$  Sagitta, the following arc lengths are obtained:

Deneb to  $\delta$ Sagitta  $29^\circ 16'$

Deneb to Albireo  $22^\circ 11'$

Albireo to  $\delta$ Sagitta  $9^\circ 50'$

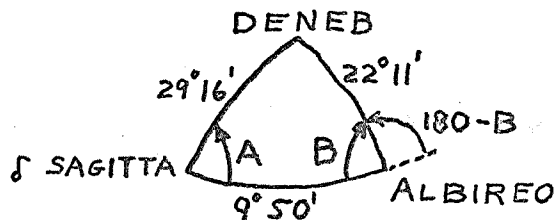


Figure 3

To find the desired angles, A and B, in Fig. 3, the spherical trigonometric cosine law is applied.

$$\cos A = \frac{\cos(22^\circ 11') - \cos(29^\circ 16') \cos(9^\circ 50')}{\sin(29^\circ 16') \sin(9^\circ 50')}$$

$$\cos B = \frac{\cos(29^\circ 16') - \cos(22^\circ 11') \cos(9^\circ 50')}{\sin(22^\circ 11') \sin(9^\circ 50')}$$

and results in:  $A = 37^\circ 14'$   $B = 39^\circ 17'$

Thus since two reference stars can be identified, a directed arc between

them can be drawn (from  $\delta$ Sagitta to Albireo, for example). From  $\delta$ Sagitta, an arc can be constructed at an angle  $A=37^{\circ}14'$  with respect to the directed arc. From Albireo, a second arc can be constructed at an angle  $180-B=180^{\circ}-39^{\circ}17'$  with respect to the directed arc. The intersection of these two arcs is the location of the desired point, in this case the pole star.

A problem with the system just described is that since the planet Mars is rotating, these two reference stars will not always be in sight of the rover at all times. This will be true for any choice of two stars. This leads to the idea of picking three identifiable stars that are separated by approximately  $120^{\circ}$  and using whichever two are in sight at a certain time. A possible choice of stars would be

	<u>Magnitude</u>	<u>Spectra</u>	<u>Right Ascension</u>	<u>Declination</u>
Albireo	3.2	K0,A0	19h 29m 37.0s	$+27^{\circ} 53' 51''$
Alcon/Mizar	2.4/4.0	A2p/A5	13h 22m 45.6s	$+55^{\circ} 04' 35''$
(Ursa Major)			13h 22m 04.0s	$+55^{\circ} 08' 18''$
Mirtak	3.1	F5,A3	3h 02m 41.1s	$+53^{\circ} 23' 39''$
(Perseus)				

The previous calculations could be carried out for this set of three stars and this would allow the pinpointing of any spot in the heavens at any time during the day.

#### C. PHYSICAL DEVICE USED FOR LOCATION OF POLE STAR AND/OR TRUE POLE

##### 1. Introduction and the Basic Star Locating/Tracking Device

Since it is desired to have the capability of locating either the pole star or the point in the heavens directly over the pole of Mars, the device for locating the binary reference stars must be capable of



maintaining the locations of these stars. This is due to the fact that a point in the sky can't be directly tracked, as a star can be. The location of the two reference stars must be known before the desired point in the sky can be located and to maintain an accurate knowledge of the position of this point, an accurate knowledge of the position of the reference must be maintained. From this point on, the star tracker capability will be considered as a necessary part of the device used to locate the reference stars.

The basic device for locating and then tracking a binary reference star is illustrated in Fig. 4.

## 2. Description of Basic Star Locating/Tracking Device

### (a) Telephoto Lens

The telephoto lens is used to gather light from all stars in a specified angular field of view (about  $2^\circ$ ), determined by the lens construction, and to direct it into a small focal plane. Instead of picking up all objects in its field of view with equal intensity, as is common in telephoto lenses, this lens is constructed in such a manner that a star in the center of its' field will be picked up as a slightly greater intensity image than a star of equal magnitude on the edge of its' field. The reason for this will be explained later.

### (b) Collimator

The collimator takes the light emerging from the telephoto lens and forms it into a tightly collimated light beam.

### (c) Prism

Upon leaving the collimator, the light beam encounters a prism made of dense flint glass\*, forming a right angle with the prism face first met ( $\alpha_1=0$ ).

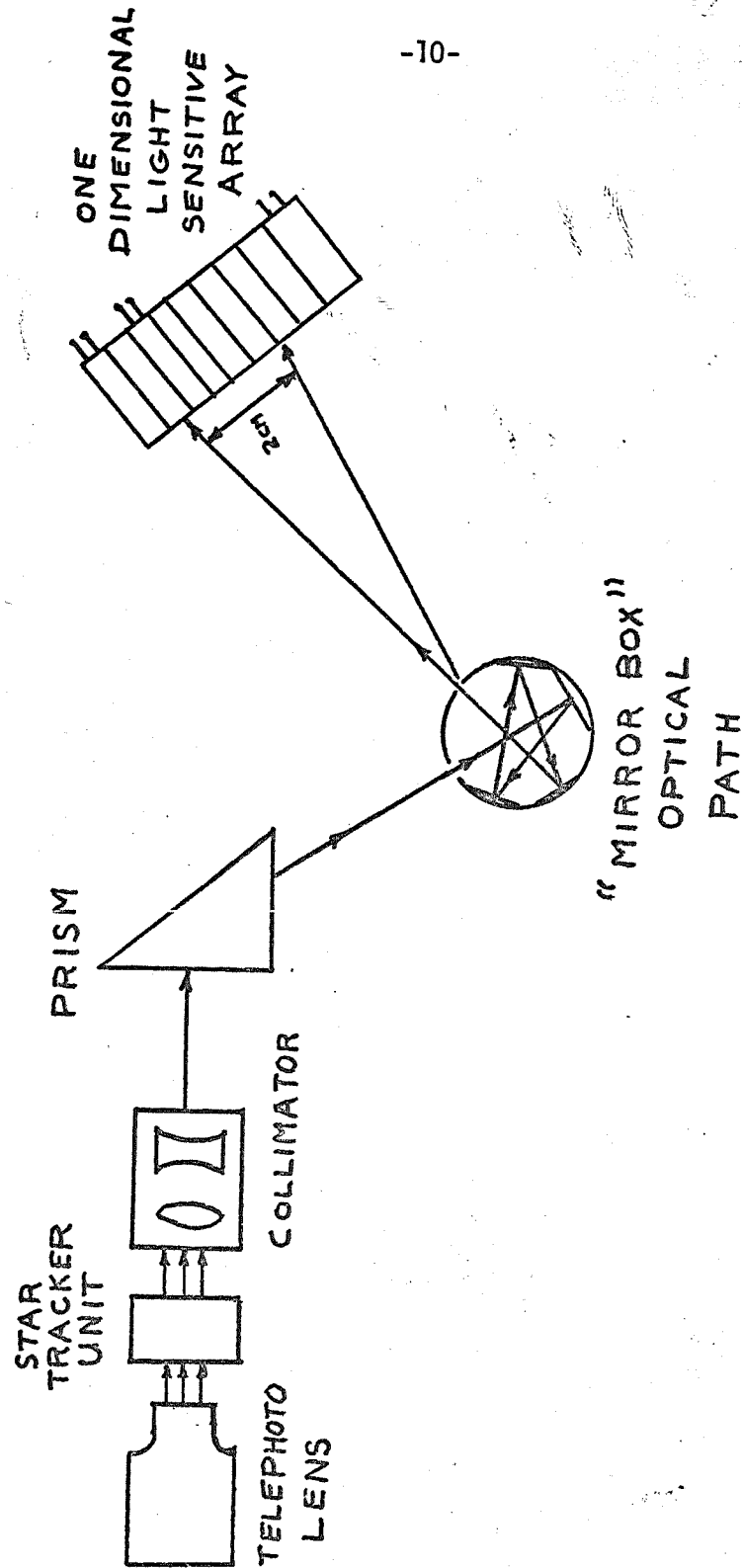


Figure 4

\*Dense Flint Optical Glass

		v=					
Cat.code (n <sub>d-1</sub> )1.00	other name	$\frac{n_{d-1}}{n_f - n_c}$	n <sub>IR</sub> (1.014μ)	n <sub>A'</sub> (0.07682μ)	n <sub>c</sub> ( 0.6563μ)	n <sub>d</sub> (.5876μ)	n <sub>f</sub> (.4861μ)
805/225	SF6	25.46	1.77517	1.78646	1.79608	1.80518	1.82771
				n <sub>h</sub> (.4047μ)	Mfg.		
				1.86428	Fish-Schurman Corp., N.Y.		

(d) Mirrored Optical Path

To obtain a distance of separation of about two centimeters between the two extreme wavelengths used above, an optical pathlength of about 0.7 meters is required. This is accomplished through the use of a mirrored box, into which the emerging beam from the prism enters. Inside the box, the diverging beams of light are reflected along an optical path of the desired length and at the end of this path they leave the box and fall upon a one dimensional array of light sensitive devices. Thus a high intensity light beam has been dispersed along a light sensitive array giving a distribution of light intensity related to the distribution of wavelength in the original beam, at a cost of having less light intensity per unit area than the original beam ( though the total light intensity is constant).

(e) Photo-detector

1. Background material - The time rate of flow of light energy is referred to as luminous flux. The luminous flux is the characteristic of radiant energy which produces visual sensation. The unit of flux is the lumen, which is the flux emitted in a unit solid angle by a uniform point source of one candela, which produces a total luminous flux of  $4\pi$  lumens [13].

In measuring stellar magnitudes photoelectrically, the flux in lumens  $L$  from a star of magnitude  $M$  which is received by a telescope having a diameter of  $d$  inches can be expressed as follows:

$$2.5 \log_{10}(L) = 7.57 - 30 + 5 \log_{10}(d) - M$$

For a telescope with a diameter of five inches and a star of magnitude three,  $L = 1.68 \times 10^{-9}$  lumens [13].

2. Comparison of Photo-Detectors - To find a photodetector which would perform satisfactorily with this magnitude of luminous flux, a study of photodetectors was performed. The approximate range of power or luminous flux for which various types of detectors are useful are indicated graphically in Fig. 6. These indicated ranges are only for guidance in picking a device and may at times be exceeded [13].

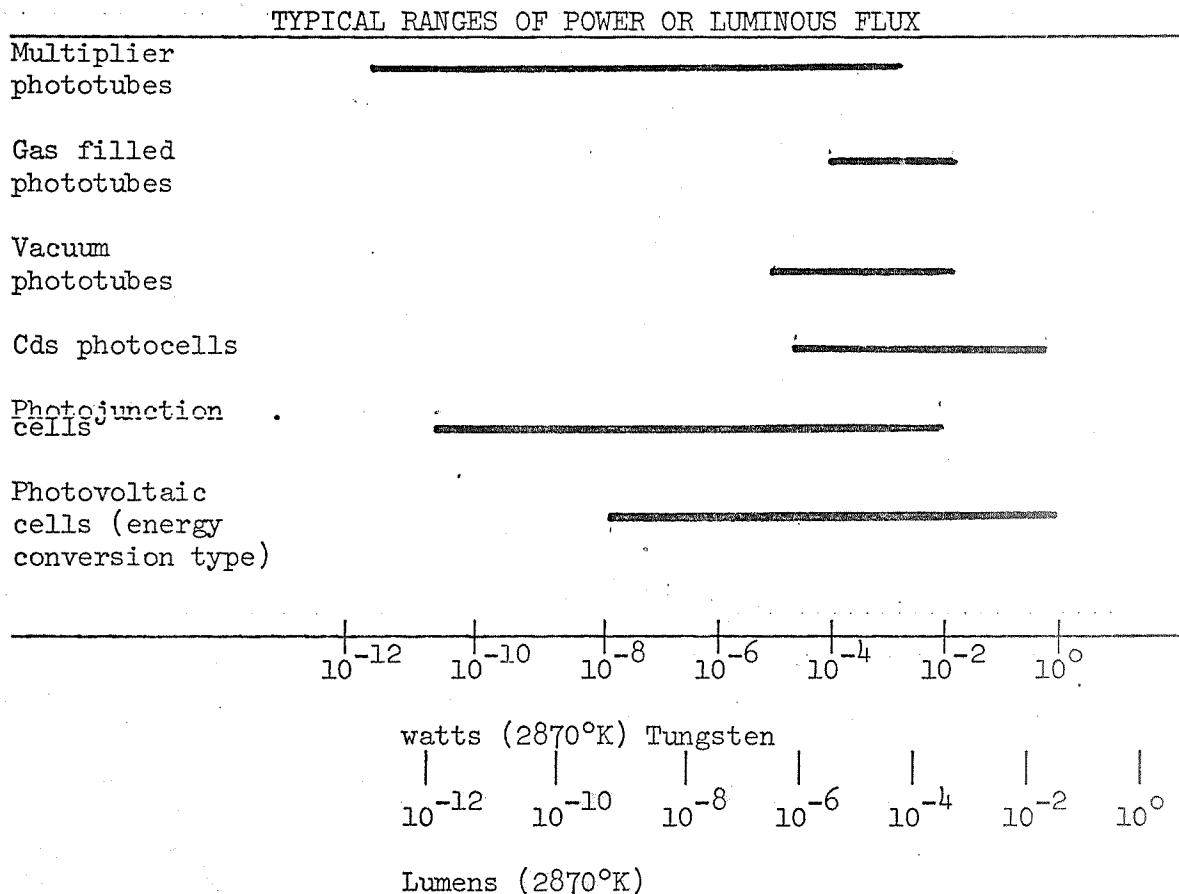


Figure 6

3. Limitations on Desired Photo-Detector - As a result of this study, it was decided to use silicon photojunction cells. Assuming an 8% loss of intensity in the telephoto lens, 4% in the collimator, 2% in the mirrored unit and 1% in the prism, the photojunction cells are still

capable of handling the job. A silicon photodiode is basically a p-n junction, exhibiting a nonohmic characteristic, as illustrated below.

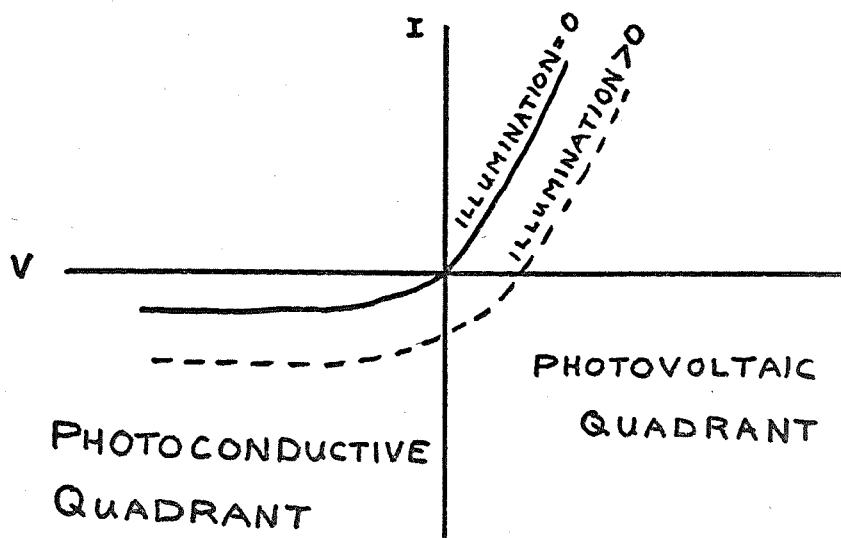


Figure 7

When light is applied to the cell, the curve shifts downward. Reverse biasing the cell, it operates as a photoconductive device and its' output is developed across a series load resistor. In photo-voltaic applications, the cell is used to convert radiant power directly into electrical power. Due to its greater sensitivity, the photoconductive mode of operation will be used [13][14][15]. Schematically, the photo-junction device connected in the photoconductive mode appears as follows:

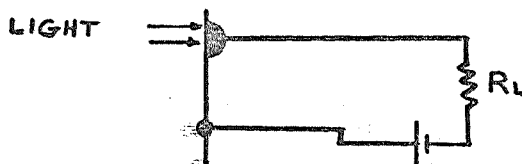


Figure 8

where the output voltage change with incident light is illustrated graphically by

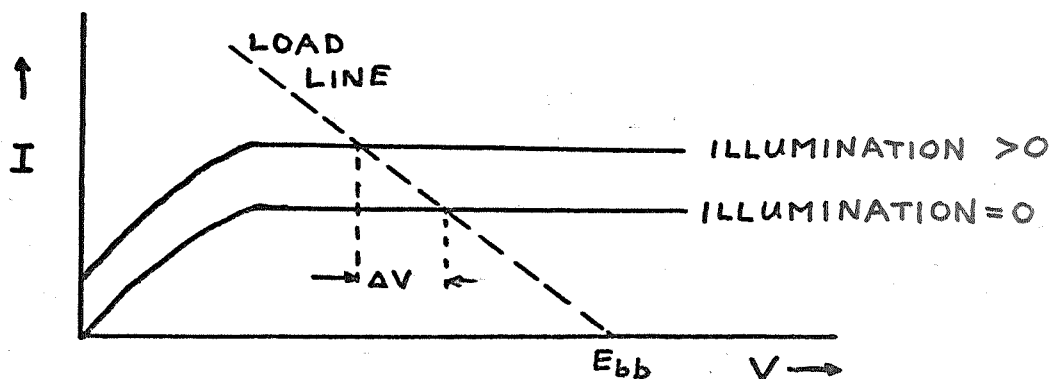


Figure 9

After a discussion with staff members in the Electrophysics Department at R.P.I., it was decided that if there was no need to worry about impulses of light and the resulting transients, i.e., the star is viewed for 1 msec or more, and the device is operated at a low current level, a one dimensional array of diodes may be fabricated having a diode width of approximately .2mil/unit. Thus a one dimensional array that is two centimeters long could contain, if desired, approximately four thousand units, with the leads coming in from above and below the array.

Assuming that an array of photojunction cells may be fabricated to operate at a level of  $10^{-12}$  lumens, the number of elements in the array is limited by the minimum amount of light required by one unit. Using the assumed losses in the optical devices,

$$\frac{(1.68 \times 10^{-9}) \times .92 \times .96 \times .98 \times .99 \text{ lumens}}{10^{-12} \text{ lumens/unit}} \approx 1450 \text{ units}$$

the limit would thus be a 1450 unit array. Since there is a spread of approximately three thousand angstroms over the range of interest, the number of units in the array is arbitrarily chosen as five hundred.

4. Operation of Photo-Detector - For a first check on the identity of the observed star, the outputs of all the array elements are summed and this sum is related to the star's magnitude. If too low a magnitude is indicated, the star is rejected and the device will begin scanning for another star. If the magnitude is great enough, the output from each array element is compared against predetermined standards corresponding to each of the desired reference stars dispersed spectra. If the observed spectra does not correspond to one of the preprogrammed spectra, the star is rejected and the device will begin scanning for another star. If the spectra does correspond to that of one of the binary reference stars, a digital signal identifying the star will be generated and a signal will be sent the star tracker unit to maintain the location of this star.

(f) Tracking of Reference Stars

The star tracking unit, upon receiving the track signal will move into place between the telephoto lens and the collimator. It will proceed to position the star at the optical center of the telephoto lens, the point where the effective intensity of the star will be the greatest. (due to the construction of the lens). This will be accomplished through the use of a segmented, light sensitive disk (CdS). Constructed of four identical wedgeshaped segments, the output of each segment is related to the intensity of light incident upon it. Through the use of a servo-mechanism system, the direction of the telescopic lens is adjusted to give equal outputs from all segments corresponding to equal light intensities on all four segments. This occurs when the star is exactly centered in the device. The scheme for controlling the servo-system is



as follows:

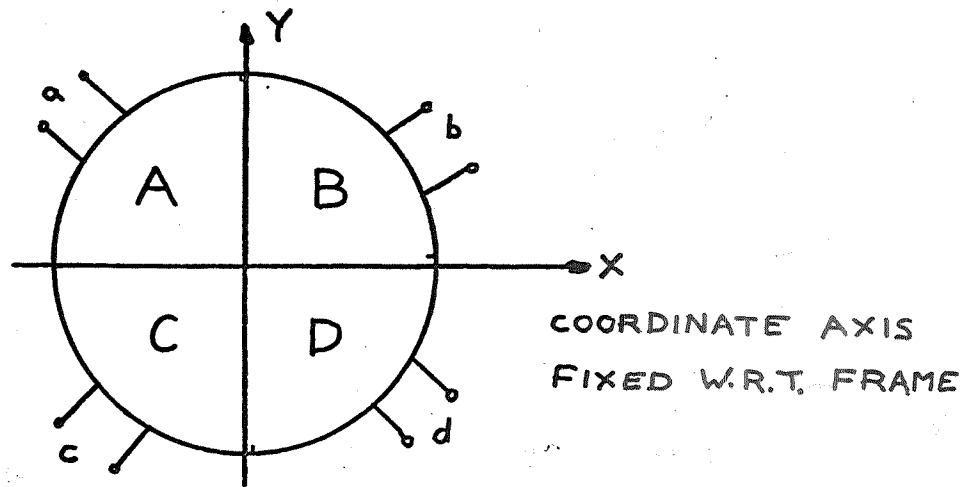


Figure 10

if $a+b > c+d$	move device in +y direction
if $a+b < c+d$	move device in -y direction
if $a+c > b+d$	move device in -x direction
if $a+c < b+d$	move device in +x direction
if $a+b=c+d, a+c=b+d$	do not move device

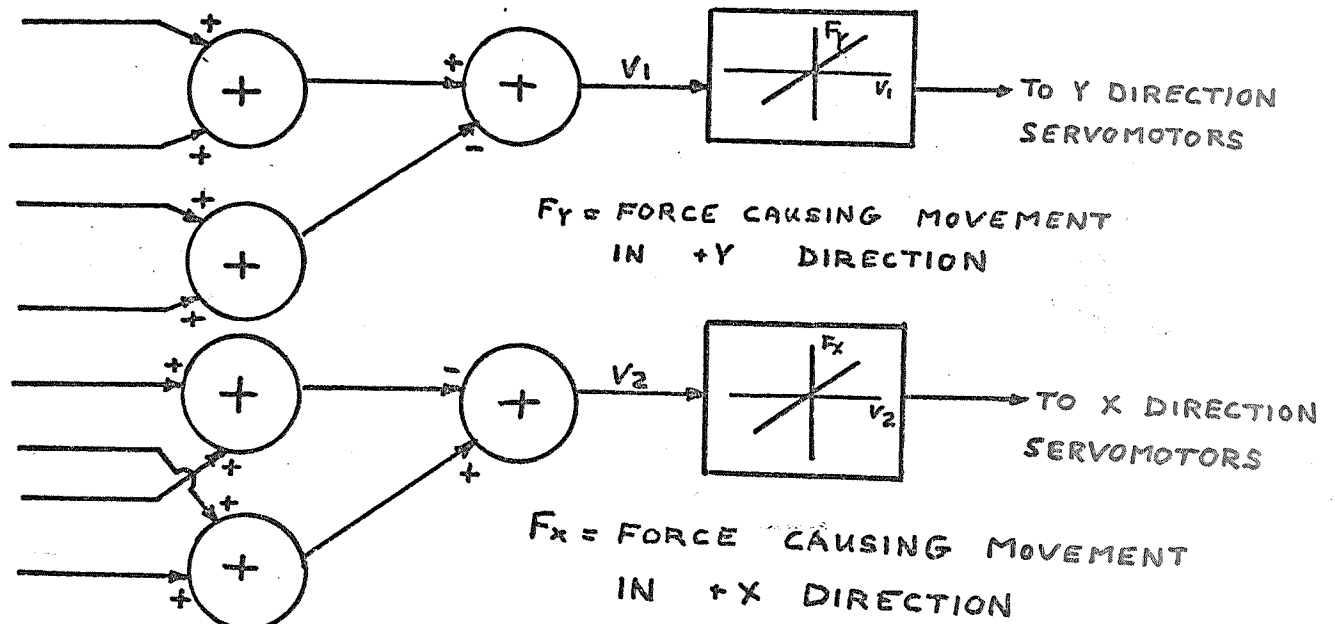


Figure 11

3. Location of Second Reference Star

Once the first reference star is located, a modified sweep path will be used to locate the second reference star. A second unit, identical with the first one just described will be positioned at an angle, with respect to the first unit, equal to the angular separation between the already located (now known) reference star and one of the other two reference stars. The second unit will sweep a constant radius circular path around the first unit. If the second reference star is located and identified, the second unit will track it and thus maintain its location. If it is not found, the angle of the first unit, with respect to the second unit, will be changed to equal the angular separation between the already found reference star and the remaining reference star. The modified sweep path will again be used, now locating and identifying the second reference star.

4. Generation of the Location of the Pole Star And/Or True Pole From Locations of Reference Stars

Having located and identified two reference stars, it is necessary to use them to locate either the pole star or the point in the heavens over the true pole. The two star locating units are mounted on a platform on the roving vehicle. They are gimbolled to move in two directions only, one direction being parallel to and the other being in a plane perpendicular to the surface of the platform, as shown in Fig.12. A fictitious reference frame, xyz, is established on the platform with the y axis running in the lengthwise direction and x axis in the widthwise direction of the platform in the platform plane and with the origin at the center of the star locating unit's gimbols. Mechanical pick-offs

measure the two angles for each unit in this fictitious frame.

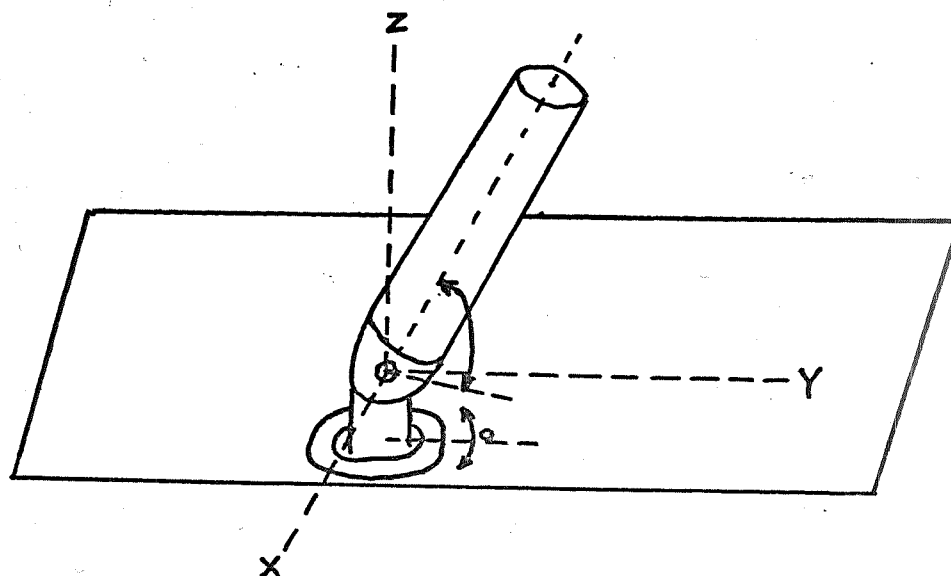


Figure 12

The zero degree point for the two gimbals in the xy plane is the positive y axis with the angle increasing in the clockwise direction and the zero degree point for the remaining gimbals is on any line in the xy plane with the angle increasing with elevation in the z direction.

In the star charts, the right ascension and declination of a star is measured with respect to a specific vernal equinox [9]. Since this is a relative measurement, the right ascension and declination of the reference stars can be measured with respect to the point in the sky the fictitious y axis is directed towards at a specific time. If this is done, the angular measurement of the gimbal in the xy plane corresponds to the right ascension and the angular measurement of the remaining gimbal corresponds to the declination of the reference star in terms of this fictitious frame. Knowing the right ascensions and declinations of the two reference stars in this fictitious reference

frame, it is possible to compute the right ascension and declination of the pole star, (or point in the heavens) with respect to this frame. Since the rest of the navigation package is also located on this platform, this gives to it the needed location of the pole star. The needed computations are better visualized by referring to Figure 13 where points 1, 2 and D refer to the locations of reference stars 1 and 2, and Deneb.

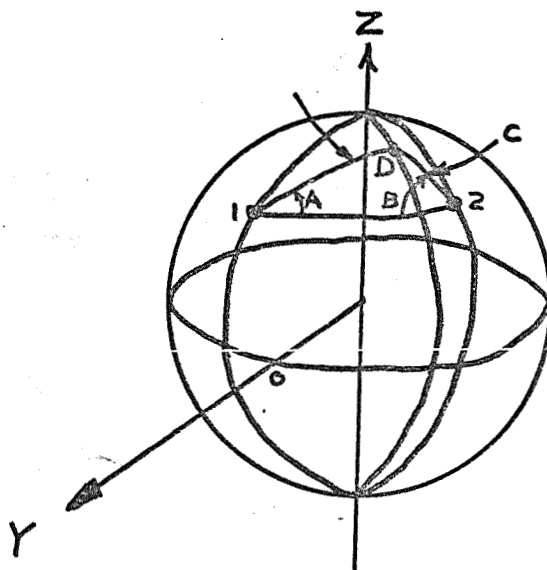


Figure 13

Through previous calculations, the included angles A, B and C are known. Using the right ascension and declination of Deneb as variables and using the law of sines from spherical trigonometry, two equations may be formed. One equation relates the arc length between points 1 and D to the arc length between points 1 and 2 and the second relates the arc length between points 2 and D to that between points one and two. This gives two equations in two unknowns. Since the identities

of reference stars one and two are uniquely known, this adds a constraint on what the right ascension and declination of Deneb can be, with respect to the ascension and declination of either reference star 1 or 2. The two equations can thus be uniquely solved for the right ascension and declination of Deneb in terms of the fictitious reference frame on the platform.

Computationally: A.B.C are known and using the definitions for angles and arc lengths used earlier in this paper

$$\cos 01 = \cos R_1 \cos d_1$$

$$\cos 0D = \cos R_d \cos d_d$$

defining  $\sin \alpha = \sin d_1 / \sin 01$

$$\sin \beta = \sin d_d / \sin 0D$$

$$\cos \gamma = \cos(\beta - \alpha)$$

results in  $\cos 1D = \cos 01 \cos 0D + \sin 01 \sin 0D \cos \gamma$

$$\cos 02 = \cos R_2 \cos d_2$$

defining  $\sin \delta = \sin d_2 / \sin 02$

$$\cos \epsilon = \cos(\delta - \beta)$$

results in  $\cos 2D = \cos 02 \cos 0D + \sin 02 \sin 0D \cos \epsilon$

$$\cos 12 = \cos 01 \cos 02 + \sin 01 \sin 02 \cos(\alpha - \delta)$$

Applying the law of sines, the two needed equations in terms of the two unknowns (the right ascension and declination of Deneb) are obtained

from  $\frac{\sin C}{\sin 12} = \frac{\sin B}{\sin 1D}$

$$\frac{\sin C}{\sin 12} = \frac{\sin A}{\sin 2D}$$

These two equations are now solved simultaneously by the vehicle's computer and the results are given to the navigation package.

## 5. Pole Star Location Error

Errors in the location of the two binary reference stars will result in an error in the generated location of the pole star and/or true pole. Considering the fabrication techniques of today, the reference stars should be located by the locating/tracking device to less than 0.1 degrees of their correct locations. Introducing these errors will result in the generation of a pole star location that should be well within the tolerances for most navigational needs. This is further discussed in Appendix III.

### D. CONCLUSION

The navigational approach to locating the pole star and/or true pole using binary reference stars is feasible, generating the pole star location exactly if the reference star locations are exact and not generating unreasonable errors in the pole star location for small errors in the reference star locations. The pattern recognition system is shown in the body of this report to be capable of operating on the light intensity emitted by the reference stars. Once the reference star is located, the method for maintaining its location and generating its coordinates in the vehicle reference frame is easily applied with a high degree of accuracy, being limited by fabrication techniques.

Looking at the total system, a pole star location error of less than one tenth (0.1) of a degree should be entirely feasible if care is taken in the construction of the system and if the platform on which it is mounted is kept reasonably stable with respect to the Martian surface.

APPENDIX I

BINARY STARS OF THE NORTHERN CELESTIAL HAMISPHERE<sup>[6]</sup>

	Magni- tude	Spectra	Right Ascension	Declination
$\tau$ Perseus	4.1	G0,A5	2h 52m 11.4s	+52°38'42"
$\gamma$ Perseus	3.1	F5,A3	3h 02m 41.1s	+53 23 39
58 Perseus	4.5	K0,A3	4 34 40.5	+41 12 24
$\zeta$ Auriga	3.9	K0,B1	5 00 26.9	+41 02 07
$\tau$ Ursa Major	4.7	F5,A5	9 08 32.7	+63 37 58
0 Leo	3.8	F5,A3	9 39 36.2	+10 01 30
113 Hercules	4.6	G0,A3	18 53 31.3	+22 36 26
0 <sup>2</sup> Cygnus	3.9	K0,B8	20 12 43.0	+46 39 09
32 Cygnus	4.2	K0,A3	20 14 34.4	+47 38 28
$\alpha$ Equalus	4.1	F8,A3	21 14 22.4	+ 5 07 37
5 Lacerta	4.6	K0,A0	22 28 19.1	+47 33 29
0 Andromeda	3.6	B5,A2p	23 00 34.9	+42 10 11
U. $\zeta$ Mizor Major{	2.4+4	A2p	13 22 45.6	+55 04 35
80 Alcore	4.0	A5	13 24 04.0	+55 08 19
Albireo $\beta$ Cygnus	3.2,5.5	K0,A0	19 29 33.0	+27 53 51
$\beta^1$ Lyra	3-4	B8p,B2p	18 49 00.5	+33 19 41
$\delta$ Sagitta	3.8	M0,A0	19 46 05.6	+18 27 42

Table I

CELESTIAL COORDINATES (1905) OF THE  
NORTH POLE OF MARS

Method	Time Base	Author	Rt.Ascension	Declination	
Position angles of polar cap	1877-86	Schiaparelli	319.90degrees	54.92deg	
	1877-86	Schiaparelli	319.6	54.2	
	1884-94	Lohse	317.18	54.35	
	1896-98	Cerulli	318.63	54.07	
	1901-05	Lowell	316.05	54.52	
	1903-09	Wirtz	312.71	53.90	
	1901-07	Lowell & Lampland	315.50	54.32	
	1901-09	Lowell	315.8-	54.4	
	1901-11	Lowell	316.61	53.95	
	1909-11	Slipher	315.5	54.0	
	1909-26	Widorn	317.0	53.5	
Longitudes of surface markings	1877--	Ashbrook	319.2	55.8	
	1952				
Latitudes of surface markings	1914-22	Pickering	315.03	51.77	
Coordinates of surface markings	1924	Trumpler	315.77	54.63	
Paths of selected surface markings	1941-50	Camichel	316.48	52.78	
Poles of satellite orbits	1877	Burton	Phobos	317.33	52.66
	1926		Deimos	315.92	52.23



APPENDIX II  
STELLAR SPECTRA<sup>[16]</sup>

All stars radiate light containing different spectral contents. A stellar spectrum is composed of a continuous spectrum overlaid by a larger or smaller number of spectral lines, absorption and emission lines. The energy distribution in a continuous spectrum is related to the effective temperature of a particular star. Maximum energy is shifted towards the shorter wavelengths with increasing temperature.

In the classification of stellar spectra, only spectral lines are used whose intensities vary markedly from spectral type to spectral type. Within a specific spectral type, stars of different luminosity classes can be distinguished by observing other lines whose intensities depend strongly on this luminosity class. The letters W, O, B, A, F, G, K, and M are used to specify the spectral type with the W-stars having the highest temperature and each successive type having a lower temperature. The W- to M-stars are referred to as the main sequence with secondary sequences consisting of the spectral types R, N and S. The characteristics of the spectra in these sequences are listed in the following table II.

DESCRIPTION OF STELLAR SPECTRA

Spectra Title	Description	TABLE II
W:	Broad emission bands, e.g. among others those of hydrogen and of neutral and ionized helium, on an intense continuous spectrum; Wolf-Rayet stars.	
O:	Absorption lines of ionized helium on an intense continuous spectrum in the shortwave region.	
B0 to B4:	Absorption lines of neutral helium and of hydrogen (Balmer lines H $\beta$ , H $\gamma$ , H $\delta$ , etc.) and of singly ionized oxygen.	
B5 to B9:	Fainter helium lines; stronger Balmer lines.	

Spectra Title	Description
A0 to A4:	Predominantly Balmer lines; some lines of ionized metals.
A5 to A9:	The intensity of the Balmer lines decreases slightly; the lines H and K of singly ionized calcium and lines of other metals are stronger.
F0 to F4:	The H and K lines are still stronger; the intensity of the Balmer lines is further reduced. Appearance of the 'G-band' in which the lines of iron, titanium and calcium lie close together.
F5 to F9:	The H and K lines are strongest; the G-band shows increased intensity.
G0 to G4:	The H and K lines are still the strongest ones, at the same time many metal lines are present; the Balmer lines are still recognizable. (The solar spectrum is of the type G1.)
G5 to G9:	The iron lines are stronger than the Balmer lines.
K0 to K4:	The continuous spectrum on the short-wave side of the K line of ionized calcium has nearly disappeared; the G-band shows maximum intensity.
K5 to K9:	Appearance similar to K0 to K4; increased occurrence of titanium oxide bands.
M:	The main feature is the titanium oxide bands; the G-band is split up into individual lines.
R:	Appearance of cyanogen and carbon monoxide bands.
N:	Spectrum similar to that of type R; the continuum on the short-wave side of $4500\text{\AA}$ has nearly disappeared; the star therefore appears red.
S:	Spectrum similar to that of M and N; bands of zirconium oxide appear.

APPENDIX III

ERROR ANALYSIS OF THE MATHEMATICAL APPROACH

FOR LOCATING THE POLE STAR

In applying the method for locating the pole star, it can be assumed that error exists in locating the two reference stars. This error will result from the combined errors created by the individual components in the two star tracking units. Since both units in the tracking mode are relatively simple, the error resulting from them should be small (less than 0.1 degrees) if care is taken in their construction.

If there is an error in the right ascension and declination of the two binary reference stars, and the mathematical procedure using the correct, predetermined, included angles A and B (see Fig. A) is followed, a location of the pole star will be generated having an erroneous right ascension and declination.

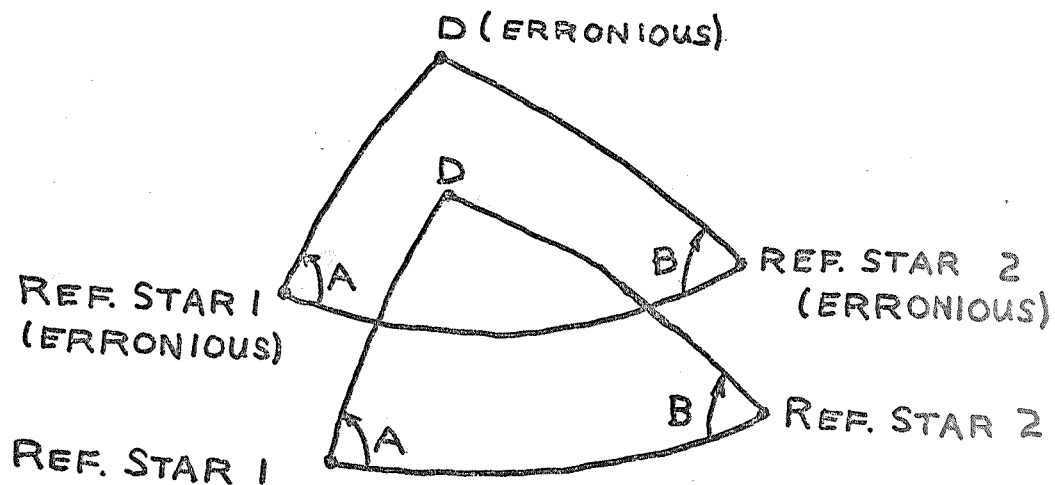


Figure A

Relations giving the errors in the pole star location are derived in the following manner:

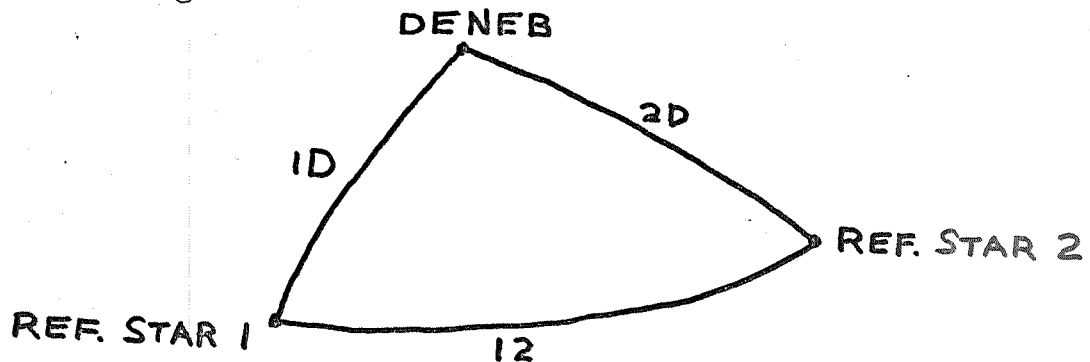


Figure B

Defining: 1D, 2D, 12 are the arc lengths between the appropriate stars.

$(R_i + r_i)$  is the erroneous right ascension of star  $i$  where  $i$  can take on values of 1, 2 or D representing reference stars 1 and 2, and Deneb.

$R_i$  is the correct right ascension of star  $i$

$r_i$  is the error in right ascension of star  $i$

$(d_i + \delta_i)$  is the erroneous declination of star  $i$

$d_i$  is the correct declination of star  $i$

$\delta_i$  is the error in declination of star  $i$

Arc length 12 may be determined using the erroneous right ascension and declination of reference stars 1 and 2

$$\begin{aligned} \cos 12 = & \cos(R_1 + r_1) \cos(d_1 + \delta_1) \cos(R_2 + r_2) \cos(d_2 + \delta_2) \\ & + [1 - \cos^2(R_1 + r_1) \cos^2(d_1 + \delta_1)]^{1/2} [1 - \cos^2(R_2 + r_2) \cos^2(d_2 + \delta_2)]^{1/2} \\ & \cdot \{\sin(d_2 + \delta_2) [1 - \cos^2(R_2 + r_2) \cos^2(d_2 + \delta_2)]^{-1/2} \\ & [1 - \sin^2(d_1 + \delta_1) \{1 - \cos^2(R_1 + r_1) \cos^2(d_1 + \delta_1)\}^{-1}]^{1/2} \\ & - [1 - \sin^2(d_2 + \delta_2) \{1 - \cos^2(R_2 + r_2) \cos^2(d_2 + \delta_2)\}^{-1}]^{1/2} \sin(d_1 + \delta_1) \end{aligned}$$

(cont'd)

$$[1-\cos^2(R_1+r_1)\cos^2(d_1+\delta_1)]^{-1/2}$$

Considering the right ascension and declination of Deneb as unknown and using the erroneous coordinates of the reference stars, two relations for arc length 1D may be derived. One relation is derived from the coordinates of reference star one and the unknown coordinates of Deneb. The second is derived using arc length 12 and the correct included angles A and B. Two relations for arc length 2D may also be derived in the same manner. The two relations for 1D must be equal, as must also be the two relations for 2D. Setting the two relations for 1D equal to each other and the two relations for 2D equal to each other, two equations in two unknowns are formed. The unknowns are the erroneous right ascension ( $R_d=R_D+r_D$ ) and declination ( $d_d=d_D+\delta_D$ ) of the pole star and they may be solved for without too much difficulty. Knowing the correct right ascension and declination, these may be subtracted from the erroneous values giving the error in right ascension and declination of the pole star. The two equations are as follows:

$$\begin{aligned} \cos 1D &= \cos(R_1+r_1)\cos(d_1+\delta_1)\cos R_d \cos d_d + [1-\cos^2(R_1+r_1)\cos^2(d_1+\delta_1)]^{1/2} \\ &\quad [1-\cos^2 R_d \cos^2 d_d]^{1/2} \{ \sin d_d [1-\cos^2 R_d \cos^2 d_d]^{-1/2} [1-\sin^2(d_1+\delta_1) \\ &\quad (1-\cos^2(R_1+r_1)\cos^2(d_1+\delta_1))^{-1}]^{1/2} - [1-\sin^2 d_d (1-\cos^2 R_d \cos^2 d_d)^{-1}]^{1/2} \\ &\quad \sin(d_1+\delta_1) [1-\cos^2(R_1+r_1)\cos^2(d_1+\delta_1)]^{-1/2} \} \\ &= \{ 1 - (\sin^2 B (1-\cos^2 12) / [1 - (\sin A \sin B \cos 12 - \cos A \cos B)^2] \}^{1/2} \end{aligned}$$

$$\begin{aligned} \cos 2D &= \cos(R_2+r_2)\cos(d_2+\delta_2)\cos R_d \cos d_d + [1-\cos^2(R_2+r_2)\cos^2(d_2+\delta_2)]^{1/2} \\ &\quad [1-\cos^2 R_d \cos^2 d_d]^{1/2} \{ \sin d_d [1-\cos^2 R_d \cos^2 d_d]^{-1/2} [1-\sin^2(d_2+\delta_2) \\ &\quad (1-\cos^2(R_2+r_2)\cos^2(d_2+\delta_2))^{-1}]^{1/2} - [1-\sin^2 d_d (1-\cos^2 R_d \cos^2 d_d)^{-1}]^{1/2} \\ &\quad \sin(d_2+\delta_2) [1-\cos^2(R_2+r_2)\cos^2(d_2+\delta_2)]^{-1/2} \} \\ &= \{ 1 - (\sin^2 A (1-\cos^2 12) / [1 - (\sin A \sin B \cos 12 - \cos A \cos B)^2] \}^{1/2} \end{aligned}$$

As an example, assume  $R_D = 0^\circ$        $d_D = 80^\circ$   
 $R_1 = 0^\circ$        $d_1 = 0^\circ$   
 $R_2 = 80^\circ$        $d_2 = 20^\circ$

and assume no error in locating reference star 1 and all the error in locating reference star 2. Angle A is 1.216441 radians and angle B is 1.447536 radians. Introducing errors in the right ascension ( $r_2$ ) and declination ( $\delta_2$ ) of reference star 2, the following errors in the right ascension ( $r_D$ ) and declination ( $d_D$ ) of Deneb result. (see Table III)

From examination of the results, it is seen that as long as the errors in the reference stars are kept small, the error in the pole star location is small. Also, for this choice of stars, the errors in the pole star location resulting from the erroneous declination of the reference star are greater than the errors introduced by an erroneous right ascension of the reference star. This indicates that the mechanical pickoff reading the declination of the reference star should be more precise than the pickoff reading right ascension.

Considering the fabrication techniques in existence today, a device for locating the pole star and/or the true pole can be constructed that will satisfy the requirements for most navigational needs.

TABLE III

ERRORS IN POLE STAR LOCATION RESULTING FROM ERRORS  
IN THE LOCATION OF REFERENCE STAR 2\*

$r_2$ (deg.)	$d_2$ (deg.)	$r_D$ (deg.)	$d_D$ (deg.)
0	0	0	0
0	0.01	0.0838	0.0002
0	0.02	0.1169	0.0004
0	0.03	0.1789	0.0006
0	0.04	0.2287	0.0008
0	0.05	0.2822	0.0009
0	0.06	0.3341	0.0011
0	0.07	0.4073	0.0013
0	0.08	0.4596	0.0014
0	0.09	0.5265	0.0015
0	0.10	0.5858	0.0016
0.10	0	0.0000	0.0342
0.10	0.01	0.0000	0.0343
0.10	0.02	0.0000	0.0345
0.10	0.03	0.1438	0.0348
0.10	0.04	0.1976	0.0349
0.10	0.05	0.2484	0.0351
0.10	0.06	0.3061	0.0353
0.10	0.07	0.3733	0.0354
0.10	0.08	0.4163	0.0355
0.10	0.09	0.4750	0.0357
0.10	0.10	0.5529	0.0358

(cont'd)

$r_2$ (deg.)	$a_2$ (deg.)	$r_D$ (deg.)	$a_D$ (deg.)
0.20	0	0.0000	0.0683
0.20	0.01	0.0000	0.0685
0.20	0.02	0.0000	0.0687
0.20	0.03	0.0838	0.0689
0.20	0.04	0.1438	0.0690
0.20	0.05	0.2146	0.0693
0.20	0.06	0.2829	0.0694
0.20	0.07	0.3451	0.0696
0.20	0.08	0.3857	0.0697
0.20	0.09	0.4613	0.0698
0.20	0.10	0.5107	0.0700
0.70	0	0.1874	0.2385

\*Coordinates for the stars are given in example statement on the preceding page.



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